be a distinct systematic difference between the separations depending on the pressure used in the source. It is conceivable that this is real. On the other hand it may be due to wide variation in intensities of the unresolved components. If we assume that the fourth and fifth components have zero intensities on the low pressure plates and theoretical intensities on the high pressure plates, the systematic difference is greatly reduced. The only conclusion to be drawn at present from this difference is that the uncertainty of the separation must be much greater than is indicated by the calculated probable errors. Our best estimate of this uncertainty may be expressed by saying that $1/\alpha$ probably lies between 137 and 138.

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Are the Formulae for the Absorption of High Energy Radiations Valid?

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In this paper we consider the discrepancies between theoretical prediction and experiment for the absorption of cosmic-ray electrons and gamma-rays. By applying a strict criterion for the validity of classical electron theory, it is possible to derive new formulae for impact and radiative energy losses of high energy electrons, which may be regarded as theoretical lower limits for these quantities, and which are in far better agreement with experiment than the formulae given by an uncritical application of quantum mechanics to these problems. These limitations on classical electron theory are consistent with those given by possible unitary classical field theories, but are more incisive than those given by the unitary theory of Born.

1. The Theoretical Formulae

THE question of the validity of the theoretical formulae for the absorption of high energy radiations has been brought to a new prominence by recent experimental and theoretical researches. On the one hand the observation of the cloud chamber tracks of cosmic rays has made it possible to extend our knowledge of the specific ionization and energy loss of electrons from particles of a few million volts on up to a few billion.[1] On the other hand two mechanisms of absorption, increasingly important at high energies, have been carefully investigated theoretically:[2] the pair production by gamma-rays, and the radiative energy losses of electrons. The question of whether the formulae derived for the probability of these processes, and the more familiar formulae for the ionization and impact energy losses of fast electrons, should hold for the very high cosmic-ray energies, has often been discussed, and has been explicitly studied by v. Weizsaecker[3] and by Williams.[4] The conclusion to which these researches have led is that the formulae should remain valid. The experiments, however, do not speak for this. We want here to reconsider the question in the light of this discrepancy.

The predictions of the theory are these: (1) The specific primary ionization of an electron (or positron) should pass through a minimum as the energy of the electron increases, and should increase slowly with the energy throughout the entire range of cosmic-ray energies. If the velocity of the electron be $v=\beta c$, then the specific ionization should vary[5] with v according to

$$(1/\beta^2)[\ln \epsilon\beta+\ln (k/\alpha)-\tfrac{1}{2}\beta^2]$$

$$\text{with} \qquad \epsilon=(1-\beta^2)^{-\frac{1}{2}}; \quad \alpha=e^2/\hbar c. \tag{1}$$

Here k is a constant of the order of 10, depending on the f-values of the atomic electrons of the matter through which the ray is passing. According to this formula one has to expect an

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[1] C. D. Anderson and S. H. Neddermeyer, International Conference on Physics, London, 1934.

[2] H. Bethe and W. Heitler, Proc. Roy. Soc. **A146**, 83 (1934).

[3] v. Weizsaecker, Zeits. f. Physik **88**, 612 (1934).

[4] E. J. Williams, Phys. Rev. **45**, 729 (1934).

[5] e.g., H. Bethe, *Handbuch der Physik*, XXIV, 1, 2nd edition, 1932.

increase of the ionization in air, which amounts to seventy percent when the energy of the electron rises from three million to a billion volts.

Often it is not the primary ionization which is observed, but the "probable" ionization:[6] the total ionization produced by the primary and by secondaries with energies less than $\epsilon' mc^2$ (in practice $\epsilon' \gg \alpha^{-2}$, $\epsilon' < 1$). This probable ionization is measured roughly by the total energy loss to secondaries of energy $< \epsilon' mc^2$, and increases with ϵ less rapidly than the primary ionization, since the increase in ionization comes chiefly from secondaries of very low energy. For the probable ionization we have instead of (1)

$$(1/\beta^2)\{\ln(\beta\epsilon) + \ln(k_1\sqrt{\epsilon'/\alpha^2}) - \tfrac{1}{2}\beta^2\}, \quad \text{(1a)}$$

with $k_1 \sim 2.5$ for hydrogen.

(2) An electron passing through matter will be accelerated by the nuclear fields, and will radiate. If σ_ν is the differential cross section of a nucleus for radiation of a quantum of energy $h\nu$, one may define a cross section for energy loss

$$\sigma = (1/\nu_0)\int_0^{\nu_0} \sigma_\nu \nu d\nu; \quad \nu_0 = \epsilon mc^2/h.$$

This has been computed by Heitler and Sauter[7] for a model in which the nuclear field is taken to be the Coulomb field of a charge Ze:

$$\sigma = 4\alpha Z^2\rho^2(\ln 2\epsilon - \tfrac{1}{3}), \text{ with } \rho = e^2/mc^2, \quad (2)$$

a result valid for $\epsilon \gg 1$. These calculations have been extended by Bethe and Heitler to the case of a nuclear field screened externally by the statistical charge distribution of the atomic electrons. With this model σ does not increase indefinitely, but approaches, for $\epsilon \rightarrow \infty$, an asymptotic value

$$4\alpha Z^2\rho^2[\ln(183/Z^{\frac{1}{3}}) + 1/18]. \quad (3)$$

For energies above 10^8 volts σ does not differ seriously from its limiting value (3). Essentially these same results have been derived by v. Weizsaecker by a method which we shall have to consider in detail.

(3) Gamma-rays of high energy will produce pairs in nuclear fields. Bethe and Heitler have carried through the calculations of the cross section for pair production in the fields of unscreened and screened nuclei, and find

$$\sigma = 4\alpha Z^2\rho^2[7/9 \ln 2\epsilon - 109/54], \quad (4)$$

$$\sigma = 4\alpha Z^2\rho^2[7/9 \ln(183/Z^{\frac{1}{3}}) - 1/54]. \quad (5)$$

All of these calculations are approximate, for the effect of the nuclear field is treated as small, and the formulae obtained are to be regarded as the first terms in a series of powers of αZ. Complete calculations which do not involve this approximation have not been made; but by using the wave functions of Furry[8] it is at least possible to see that the error involved does not become progressive as the energy increases, that, for example, no terms of the form $\alpha Z\epsilon$ occur; and this conclusion, as Williams has observed, follows also from the argument of v. Weizsaecker. The formulae (4) and (5) have thus to be regarded as legitimate approximations for small Z, no matter how great the energy. In the case of pair production σ is known to be quite accurately proportional to Z^2 for gamma-rays of energy 2.6 and 5.4 million volts;[9] and it appears from this that one may expect (5) to hold without serious error even for heavy elements.

2. The Experimental Evidence[10]

According to (3), a beam of high energy electrons should have a good part of its energy converted into gamma-radiation in a centimeter of lead; in an equal distance this gamma-ray will be largely reconverted into pairs. The two mechanisms together therefore furnish a very rapid mechanism for the degradation and absorption of electrons, positrons or gamma-rays, an absorption which deviates strongly from a mass absorption law. It is therefore possible to do justice to the great penetration of the cosmic rays only by admitting that the formulae are wrong, or by postulating some other and less absorbable component of the rays to account for their penetration.

Other arguments lead to this same alternative. Thus it is possible to observe the ionization of

[6] E. J. Williams, Proc. Roy. Soc. **A135**, 108 (1932).
[7] Reference 2.

[8] W. H. Furry, Phys. Rev. **46**, 391 (1934).
[9] E. McMillan, Phys. Rev. **46**, 868 (1934).
[10] Such clarity as there is in this account of the experimental situation I owe entirely to Dr. Anderson and Mr. Neddermeyer, who have with great patience explained to me just what the evidence is, what it indicates, and how little it proves.

cosmic-ray tracks in cloud chambers. This has been done by Anderson and Neddermeyer and by Kunze,[11] who fail to find evidence for the increase in ionization with ϵ predicted by (1). According to Dr. Anderson, estimates of two kinds have been made:

(a) Estimates of the density of thin tracks, which should give the *primary* ionization, for which (1) holds. Although a seventy percent difference in density for low energy β-particles could be detected, no difference could be detected for tracks varying from a few million to a few billion volts.

(b) Actual counts on diffuse tracks. Here it is the total ionization produced by primary for secondaries of energy below some value $\bar{E}$ which is measured. $\bar{E}$ is determined by the fact that when an energetic secondary is curled up in the magnetic field, it makes the separate counting of the drops impossible. $\bar{E}$ probably lies between 10^3 and 10^5 volts. For this "probable" ionization we have instead of (1) to use (1a). Here again there is no large increase in ionization with ϵ. A small increase might still escape detection.

It would thus seem necessary to say, either that the increase of ionization predicted by (1) or (1a) does not occur, or that all of the high energy tracks by Anderson are made by protons. This second alternative, which has been seriously advocated by Williams,[4] meets with the difficulty that there are tracks (with an $H\rho$ corresponding to a 5×10^8 volt electron, for instance) for which one would expect an ionization observably greater than the minimum, whether they are made by electrons or protons. But the uncertainties in the ionization observations do not make it possible to exclude the possibility of protons completely.

Anderson and Neddermeyer[1] have made studies of the energy losses in lead plates. For tracks of not too high energy ($\sim3\times10^8$ volts), the energy loss can be directly measured. There is good evidence for large energy losses, which are almost certainly radiative. The losses are smaller than one would expect from (3). But the number of tracks is small; large fluctuations are to be expected; and it is not certain, though it is probable, that the formula (3) gives too high a result.

With higher energy tracks, where energy losses are not directly measurable, one can still conclude that, if the tracks are made by electrons (and positrons) both (3) and (5) cannot be right, since one does not observe at all that multiplication of tracks by gamma-radiation and pair production which (3) and (5) would predict.

Against the hypothesis that these high energy tracks are made by protons, positive and negative, there are two further arguments. In the energy range where one can unambiguously distinguish between electronic and protonic mass, protons are an extreme rarity, and although positive and negative curvatures occur with about equal frequency for the high energy tracks, no definitely recognizable negative protonic tracks have been seen. The second argument concerns the production of high energy secondary electrons. The number and distribution of these corresponds to what we should expect for primaries of electronic mass, and can hardly have been produced by protons of the observed distribution in $H\rho$. It does not seem likely that protons are important in the energetic part of the cosmic radiation.

Little evidence exists for the validity of the theoretical formulae for pair production by gamma-rays of very high energy. The theoretical formulae hold quite well up to energies of 10^7 volts, but beyond that there are no definite tests of the formulae. Gilbert[12] has, however, measured the absorption coefficient of the shower producing components of the cosmic rays. This absorption follows the Z^2 law; and the radiation is probably a gamma-radiation. The total absorption found by Gilbert is, however, only one-fourth of that to be expected from (5) for single pair production alone.

It is with this experimental evidence in mind that we wish to reexamine the question of the validity of the theoretical formulae. We shall see that when we restrict ourselves to those contributions to the formulae where the applicability of theory cannot be held in doubt, we obtain results to be regarded as theoretical lower limits for the probability of the processes in question—which in every case differ radically from the corresponding formulae of Section 1. For radia-

[11] P. Kunze, Zeits. f. Physik **83**, 1 (1933).

[12] C. W. Gilbert, Proc. Roy. Soc. **A144**, 559 (1934).

tive losses, for the probable total ionization, for pair production by gamma-rays, the modifications in the formulae appear to resolve satisfactorily the discrepancies with experiments. This is probably not true in the case of primary ionization, for which the modified formulae still call for a detectable increase in the range of cosmic-ray energies.

3. The Limitations of Classical Electron Theory

The origin of the critique of the theoretical formulae lies in classical electron theory. The domain of applicability of this theory is limited to problems in which an unambiguous separation of the field of the electron itself and the external field acting on it is possible: in which, that is, the effect of the proper field is with good approximation given by the inertia of the electron, and in which the radiative reaction of the electron may be treated as a small correction. Thus we may consider the expansion given by Lorentz[13] for the proper force of an electron, taken momentarily at rest, and considered as a distribution of charge, spherically symmetric, and limited to a region of order of magnitude $\rho=e^2/mc^2$:

$$F=m\ddot{x}+2e^2\dddot{x}/3c^3+O(e^2\rho\ddddot{x}/c^4). \qquad (6)$$

The condition that the terms in this series decrease rapidly is then that

$$\dddot{x}\rho/\ddot{x}c\ll 1;\quad \ddddot{x}\rho/\dddot{x}c\ll 1,\ \cdots$$

etc. The frequencies of the motion of the electron must therefore be small compared to $\bar{\nu}=mc^3/e^2$. When the external forces are in this sense slowly varying, classical electron theory can be unambiguously applied.

This condition: that the radiative reaction be small compared to the inertial reaction—and thus the external ponderomotive force—does not depend on the choice of reference system; in fact the usual method of computing the radiative forces in a system in which the electron is not at rest is to transform $2e^2/3c^3\dddot{x}$ by a Lorentz transformation, under which all terms of (6) transform similarly. But only in a coordinate system in which the electron is substantially at rest $(1-\beta^2\sim 1)$ can the criterion for the applicability of electron theory be put simply as the condition that the fields acting on the electron shall not vary much in a time $\bar{\tau}=1/\bar{\nu}$.

Since the formalism of the Dirac electron theory and the quantum theory of the field may be regarded as a natural quantum theoretic generalization of the dualistic classical electron theory, one may expect that this formalism too will fail in the same region as its classical counterpart. The fact that a relativistic quantum theory is possible at all depends then essentially upon the smallness of α, which gives the relative magnitude of successive terms in (6) for the frequencies $\sim mc^2/h$ characteristic of relativistic electron theory. Since, in the problem of the energy losses and radiation of very high energy electrons, energies corresponding to frequencies $\geq\bar{\nu}$ necessarily occur, the question of the validity of the theoretical formulae requires investigation.

4. Application to the Classical Theory of Energy Loss[14]

One may treat the ionization and energy loss of a fast electron by computing the probability of transition induced in the atomic systems by the field, calculated classically, of the (undeflected) primary; for all impacts in which the momentum transfer is small compared to the primary momentum this treatment is fully justified; and it is the probability of these impacts which can be observed by studying the primary and 'probable' ionization of the electron tracks. The components of the primary field responsible for this ionization are low frequency components, for which $\nu\ll\bar{\nu}$; the secondary electron, in these impacts, never attains velocities very close to that of light, and if only these low frequency components acted on the electron, there could be no question

[13] H. A. Lorentz, *Theory of Electrons*, p. 252.

[14] The question of the energy losses of very energetic electrons has been much considered by Swann, who has also emphasized that with increasing ϵ the radiative forces may increase enormously. The present treatment differs from Swann's in two essential points: (a) Swann concludes that when the radiation computed classically would be equal to the energy transferred, no transfer at all will occur, whereas we argue that the magnitude of the radiation reaction merely makes classical electron theory inapplicable. (b) Swann assumes that for large ϵ the radiative reactions are large for all impacts, whereas we find this true only for impacts with parameter $p<\bar{p}=\rho\epsilon$. This is why we find a finite constant lower limit for the energy loss, instead of concluding, as does Swann, that it should vanish as $\epsilon\rightarrow\infty$. We do not believe that the *vanishing* of the energy loss can be justified by any electrodynamics.

of the validity of the theoretical formulae. It is substantially this argument which has so often led to the conclusion that the formulae should hold.[15] The cogency of this argument can, however, be questioned. The argument may be formulated in this way: *Assume* the validity of theory to describe the reaction of the electron to all components, high as well as low, to the field, then we can show that the high frequency components contribute nothing to the probability of the processes—small energy transfers—in which we are interested. Then for the low frequency components alone there is no question of the validity of theory. But to establish this it is necessary to assume the validity of the theory also for the high frequency components, and this assumption cannot be justified. The condition for the rapid convergence of (6) is a condition on the total motion of the electron, and thus on the whole external field acting on it, and we must be prepared to find that the ponderomotive force acting on the electron cannot, when rapidly varying fields are involved, be taken simply as the sum of the forces exerted by the separate Fourier components. It is in this point that we differ from v. Weizsaecker and from Williams; and it is only by insisting on this that we can understand at all why the theoretical formulae can fail.

We are here making a distinction, which in the domain of classical electron theory does not need to be made, between the external field strengths computed by classical theory, and the ponderomotive force, which is of the same general character as that developed by Born[16] in his modified unitary electrodynamics. Such a distinction is possible only in a theory in which the field equations are not linear, since for a linear theory it would follow from the conservation laws that the ponderomotive force of the sum of two fields is the sum of the ponderomotive forces of the separate fields. The existence of such nonlinearities seems, however, inevitable in any theory which would account for the specific stability of the electron; and it may be remarked that the theory of the positron, even in its present incomplete form, involves such nonlinearities for the field equations.

In the following discussion we shall then suppose—in distinction to v. Weizsaecker and Williams—that whenever high frequency components are present in the external field with an amplitude comparable to that of the low frequency components, the application of electron theory becomes dubious. To the question of the application of Born's electrodynamics to these problems we shall return in Section 7.

The normal component of the electric intensity in the field of the primary which is responsible for the greater part of the ionization, is given by the Fourier resolution:

$$\mathcal{E}^{\perp}=\int e^{i\nu t}\mathcal{E}_{\nu}{}^{\perp}d\nu, \quad \text{with}$$

$$\mathcal{E}_{\nu}{}^{\perp}=\frac{\pi i e\nu}{2\epsilon v^2}H_1{}^{(1)}(\nu p i/\epsilon v)\cdot e^{-i\nu z/v}. \qquad (7)$$

Here p, the impact parameter, is the distance from the track, z is measured along the track and $=0$ for $t=0$, and again $\epsilon=(1-\beta^2)^{-\frac{1}{2}}$, and v is the primary velocity. The components of frequency $\nu\geq\bar{\nu}$ are large near the track, and begin to fall off rapidly as $p>\bar{p}=\epsilon v/\nu\sim\epsilon\rho$. $\mathcal{E}^{\perp}{}_{\text{maximum}}$ at $\bar{p}$ is $m^2c^5/e^3\epsilon v$, and thus for $\epsilon\gg 1$, the field is always weak. We may, however, expect that for impacts in which the field within $\bar{p}$ is of importance the theoretical calculations can give totally wrong results. In a purely classical calculation of the energy loss, the *omission* of such impacts has the effect of introducing a lower limit $\bar{p}$ for the impact parameter, and thus gives an energy loss which does not increase with ϵ; instead of the classical formula of Bohr: energy losses $<mc^2$

$$\sim(4\pi e^4/mv^2)\ln(\epsilon m v^3/e^2\omega), \qquad (8)$$

(ω the resonance frequency of the atomic electrons), we obtain

$$\sim(4\pi e^4/mv^2)\ln(mv^3/e^2\omega). \qquad (9)$$

The increase in energy loss which comes from the equatorial flattening of the field, and the consequent increase in the upper limit of the impact parameter ($p_{\text{max.}}\sim v\epsilon/\omega$), is compensated by the decrease in close impacts. The impacts which are excluded by taking $\bar{p}$ as a lower limit for the impact parameter involve relatively large energy transfers; but for $\epsilon\sim 100$ transfers of

[15] Reference 6; J. F. Carlson and J. R. Oppenheimer, Phys. Rev. **41**, 763 (1932).

[16] M. Born, Proc. Roy. Soc. **A143**, 410 (1934); M. Born and L. Infeld, Proc. Roy. Soc. **A144**, 425 (1934).

energy of the order of 20 volts are being excluded. We might therefore suppose that for primary energies $>5\times10^7$ volts, the observed ionization will no longer increase with energy. To the difficulties in extending these considerations to a quantum theoretic calculation of ionization we shall return later. In the problem of radiative losses these complications do not arise.

5. Application to Radiative Losses

In the treatment of radiative losses one considers the probability of radiation of an electron (or positron) when it is accelerated in the screened field of a nucleus—a field given (very roughly) by the potential

$$V = Ze\{1/r - 1/r_0\} \quad \text{for} \quad r<r_0$$
$$=0 \quad \text{for} \quad r>r_0$$
with $\quad r_0 = \hbar^2/me^2Z^{\frac{1}{3}}.$

The frequencies of the radiated energy are of the order $\epsilon mc^2/h$; the velocity of the electron is large, and here again it is not at once clear whether classical electron theory should be applicable. To simplify the consideration of this question, v. Weizsaecker has considered the problem in another coordinate system: that in which the impinging electron is at rest. This electron is now accelerated by the field of the passing nucleus, and will radiate. One needs, however, in this coordinate system to consider frequencies for the emitted radiation which are of the order of mc^2/h; and in this system, for such radiative processes, the electron never attains an energy very large compared to mc^2. The situation is thus quite analogous to that in the problem of ionization: the components of the field of the passing nucleus which one needs to consider are $\ll\bar{\nu}$, and this in a coordinate system where the electron does not attain a velocity very close to c. Here again, in the classical treatment of the problem, there are impacts for which the field acting on the electron varies rapidly in a time $\bar{\tau}=1/\bar{\nu}$, and for which therefore we must call in question the validity of the electron theoretic treatment.

v. Weizsaecker has in fact shown that one may give a semi-classical treatment of the problem, for $\epsilon\to\infty$, which leads to (2) and (3). In this treatment one introduces again an impact parameter p, considers the radiation for an electron initially at p, and integrates over p. This treatment can be justified for such values of p that the field of the nucleus is there varying little over a wave packet which is large enough to permit a fair definition of the momentum change of the electron during this impact; such wave packets are large compared to $\hbar/mc$; and one thus concludes that for values of $p\gg h/mc$ the method may be used. For $p<h/mc$ the field varies rapidly; and v. Weizsaecker shows that for such impacts we may expect little radiation. Thus $\hbar/mc$ is roughly the lower limit of the impact parameter. For an unscreened nucleus the percentage error introduced by the necessary vagueness of $p_{\text{min.}}$ vanishes with $\epsilon\to\infty$; for a screened nucleus it does not, but remains of the order $\hbar/mcr_0$. This is because, for an unscreened nucleus, the outer limit of the impact parameter, determined by the condition that the impact time be not too long compared to h/mc^2, increases with ϵ; whereas for the screened nucleus it remains $=r_0$.

For $\epsilon\gg1$, $p>\hbar/mc$, the field of the nucleus can now be represented with good approximation as the superposition of plane electromagnetic waves traveling parallel to the nucleus; the amplitude of these waves is given, from (7), by

$$\mathcal{E}_\nu \sim (\pi i\nu Ze/2\epsilon c^2)H_1^{(1)}(i\nu p/\epsilon c).$$

The radiative losses can thus be regarded as arising from the scattering of these waves; and for the treatment of this v. Weizsaecker uses the formula of Klein-Nishina—the result is

$$\sigma\sim 4\alpha Z^2\rho^2[\ln p_{\text{max.}} - \ln p_{\text{min.}}]$$
$$\sim 4\alpha Z^2\rho^2 \ln\epsilon \qquad \text{for no screening} \quad (3a)$$
$$\sim 4\alpha Z^2\rho^2 \ln(1/\alpha Z^{\frac{1}{3}}) \qquad \text{screening.} \quad (4a)$$

The justification for regarding these formulae as valid for large ϵ v. Weizsaecker finds in the circumstance that only components of the nuclear field and the radiation field for which $\nu\ll\bar{\nu}$ play a part in these results.

We have, however, to remember that for impacts for which $p<\bar{p}=\epsilon\rho$, the impact time is short compared to $\bar{\tau}$, and frequencies $>\bar{\nu}$ appear in the field acting on the electron. They do not contribute *directly* to the probability of radiative losses, but their presence makes the application of electron theory and the use of the superposition principle for the force on the electron questionable. If we omit altogether the contribution of these impacts, we have again to introduce a new lower limit for p, $\bar{p}=\epsilon\rho$. For $\epsilon>137$, $\bar{p}>\hbar/mc$, and we find smaller radiative losses than those given by (3a) and (4a):

$$\sigma\sim4\alpha Z^2\rho^2\ln(1/\alpha)\qquad\text{no screening,}\qquad(3\text{b})$$

$$\left.\begin{array}{l}\sim4\alpha Z^2\rho^2\ln(1/\alpha^2Z^{\frac{1}{3}}\epsilon)\\ \sim0\quad\text{for}\quad\epsilon>\sim\alpha^{-2}Z^{-\frac{1}{3}}\end{array}\right\}\text{screening.}\qquad(4\text{b})$$

6. Discussion: Application to Primary Ionization

Formula (9) for the ionization energy losses, and (4b) for radiative losses (and presumably γ-ray pair production), have been derived by neglecting all contributions from those impacts to which classical electron theory may not certainly be applied; they thus give us lower limits for the effects to be expected in fact, since it is possible (and for some problems certain) that *some* contribution will come from these 'fast' impacts which cannot at present be rigorously treated.

These lower limits (4b) and (9) differ significantly from (4a), (8), the theoretical formulae obtained by applying present theory to all impacts. For in (9) there is no increase with ϵ of the low energy ionization energy losses. According to (4b), moreover, the probability of radiative losses (and pair production by γ-rays) begins to *decrease* for $\epsilon>137$, and vanishes altogether for $\epsilon\sim(137)^2Z^{-\frac{1}{3}}$, or 10^9–10^{10} v. In spite of the rough nature of these conclusions, and the tentativeness of the experimental results, we may thus say that the discrepancies between theory and experiment which appear to exist when we use the theoretical formulae of Section 1 disappear when we leave out of consideration those processes where the application of present theory is dubious.

In the classical calculation of energy transfer, we have seen that the limitation on the impact parameter $p>\bar{p}$ leaves out all impacts in which large energies are transferred to the secondary. The number and distribution of the secondaries with energies $E'\gg mc^2$, $E'\ll\epsilon mc^2$, has been studied by Anderson and Neddermeyer.[1] They find good agreement with the classical formula for the probability of production of such secondaries:

$$\sigma dE'=(\pi e^4/mc^2)(dE'/E'^2).\qquad(10)$$

(In the range investigated the interchange terms by which the quantum theoretical formulae differ from (10) are negligible.) The number of secondaries is small, and the observed energy has to be corrected for energy loss; nevertheless, these experiments give no evidence of a discrepancy here between theory and experiment, and show that high energy secondaries are produced, and with frequency that can hardly be less than that predicted by a factor of two. These secondaries are produced in impacts to which we should not expect electron theory to apply; and the approximate validity of (10) cannot be justified from our point of view. What the experiments themselves seem to show is that the theoretical predictions for the number of high energy secondaries are not more seriously in error than those for low energy secondaries.

The classical treatment of small energy losses cannot be justified quantum theoretically. For one cannot make wave packets which at the same time define precisely enough the momentum of the electron ($\Delta P<\alpha mc$) and over which the field varies relatively little, except for values of the impact parameter $p>\hbar/mc\alpha$; and impacts for smaller p contribute essentially to this energy loss. One can, however, formally obtain the correct quantum theoretic answer for the energy by introducing as a lower limit for p not ρ but $\hbar/mc$.[17] If we do this but introduce $\bar{p}$ as a further lower limit, we find that the low energy losses ($E'<mc^2$) increase with ϵ till $\epsilon\sim137$, and then remain constant.

When $\epsilon\gg\alpha^{-2}$, $\bar{p}\gg\hbar/mc\alpha$, classical calculations can be made for all impacts for which $p>\bar{p}$, since in this region the problem of energy transfers can be treated as a pure dispersion problem. If now we again take $\bar{p}$ as a lower limit for the impact parameter, we find that the primary ionization approaches, for $\epsilon\rightarrow\infty$, a finite limit, and in place of (1) obtain:

$$\sim\ln(\alpha^{-3})\quad\text{for}\quad\epsilon\gg\alpha^{-2}.\qquad(11)$$

This gives a primary ionization about 70 percent greater than the minimum value of (1). The corresponding result, for $\epsilon\rightarrow\infty$, for the total energy loss $<mc^2$, is given essentially by (9).

[17] F. Bloch, Ann. d. Physik **16**, 285 (1933).

This limiting value is roughly equal to the minimum value of the classical expression for this energy loss, and is about 25 percent greater than the minimum of the corresponding quantum theoretic value. These results are again to be regarded as lower limits, for $\epsilon\to\infty$, since in their derivation impacts with $p<\bar{p}$ have been omitted altogether.

To find the course of this increase of ionization with ϵ, we should have to treat wave packets so large that in a part the field was rapidly varying, and in the rest slowly varying. A simple but hardly adequate way to do this is to set the field of the primary zero within $\bar{p}$; this procedure in the classical calculation leads of course to (9). If we do this we find that for $\epsilon\ll\alpha^{-2}$, formula (1) should hold.

These conclusions help somewhat to mitigate, but do not resolve, the discrepancies between theory and the cloud chamber observations of ionization. In spite of the qualitative character of (11), we think it certain, both that electrons of arbitrarily high energy will give a primary ionization measurably greater than the minimum value, and that for large ϵ the increase predicted by (1) cannot be regarded as theoretically established.

7. Relation to Classical Unitary Electron Theories

In Section 3 we have formulated a condition for the validity of electron theory: that the successive terms in the Lorentz expansion (6) should diminish rapidly, that in particular, the radiative reaction should be small compared to the inertial reaction. The ground for the necessity of this limitation is that the stability of the electron itself is not to be understood on the basis of Maxwellian electrodynamics: non-Maxwellian forces must be assumed to account for the stability; of their nature, apart from this, we know nothing; and it is therefore not possible to take the reaction of the electron to these forces into account in detail; we have to confine ourselves to those problems in which the effect of these forces is given essentially by the inertial reaction of the electron which must then for stability be equal to the external ponderomotive force. In any classical theory which accounts for the electron's stability, the limitations we have discussed could be removed.

Recently Born has proposed a modification of Maxwellian electrodynamics in which the electron itself appears as a possible (if not unique) singular solution of the field equations of finite energy $=mc^2$. When the electron is subjected to an external field, its motion can be deduced from the conservation laws for energy and momentum, which are to hold in spite of the fact that the field equations, from which they may in general be deduced, fail to hold along some world line—the electron's path. One has thus a consistent classical theory which gives a specific answer even where the earlier electron theory could not be applied. What does this theory give for the problems of ionization and radiative loss we are here considering?

When the fields acting on the electron are weak, $|F|\ll\bar{F}=m^2c^4/e^3$, and when they vary slowly (for the electron nearly at rest $\nu\ll\bar{\nu}$) then this motion agrees with that given by electron theory. When the electron (nearly at rest) is acted on by a disturbance whose frequency grows large compared to $\bar{\nu}$, then, as Born has shown, the reaction of the electron is in general much smaller than that computed from the Lorentz Force. And when the external fields are of the order $\bar{F}$, the treatment of these fields as small perturbations breaks down, and we may again expect deviations from electron theoretic formulas.

It is with the latter condition that we are concerned, since, as we have seen, the frequencies in the field which are directly involved in energy loss and radiation are low, and deviations from classical electron theory are to be expected only if the superposition principle for the ponderomotive force breaks down. The external fields become comparable to the proper field $\bar{F}=m^2c^4/e^3$ for impact parameter $p\sim\rho\epsilon^{\frac{1}{2}}$, for impact energy losses, and for $p\sim\rho(Z\epsilon)^{\frac{1}{2}}$ for radiative losses. The theory of Born thus does not give as strong a limitation on the validity of classical theory as we have used: $\bar{p}\sim\rho\epsilon$.

This situation is, however, not intrinsic to a classical unitary electron theory, and depends upon the fact that the Lagrangian of Born's theory involves the field strengths, but not their derivatives. Consider for instance the Lagrangian $L=\frac{1}{2}\psi(1+\rho^2\varphi/\psi)^{-1}$ with $\psi=\frac{1}{2}F^{\mu\nu}F_{\mu\nu}$, and

$$\varphi=\tfrac{1}{2}(\partial F^{\mu\nu}/\partial x^\alpha)g^{\alpha\beta}(\partial F_{\mu\nu}/\partial x^\beta).$$

The field equations of this Lagrangian are of the 3rd order—and nonlinear. They have a static spherically symmetric singular solution, of finite energy, which reduces for $r\gg\rho$ to a Coulomb field. For this Lagrangian the superposition principle breaks down for

$$p\sim\rho\epsilon^{\frac{2}{3}};\quad p\sim\rho Z^{\frac{1}{3}}\epsilon^{\frac{2}{3}},$$

for impact and radiative losses respectively. More generally, if L involves derivatives of the fields of order n, and gives an electron of finite energy as a singular solution, then the superposition principle breaks down for

$$p_{\text{imp.}}\sim\rho\epsilon^{(n+1)/(n+2)},\quad p_{\text{rad.}}\sim\rho Z^{1/n+2}\epsilon^{(n+1)/(n+2)}.$$

The $\bar{p}=\rho\epsilon$ which we have used as a limit for classical theory in this paper is thus given by a unitary field theory whose field equations are integral equations, for which $n\rightarrow\infty$.

We adduce these considerations, not because we believe that the solution to the problem of electronic stability lies in a theory of this type, but because they show that there is, even in classical theory, no inconsistency in the criteria we have used for the validity of electron theory. The discrepancies between theoretical prediction and the experiments can thus be understood on a purely classical basis.

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The Disintegration of the Nuclei of Light Atoms by Neutrons

II. Neon, Fluorine and Carbon

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Neon and fluorine. Quantitative information is presented on the disintegration, by capture of a neutron, of 11 nuclei of neon and 13 of fluorine. The reactions are considered to be:

$${}^{20}_{10}\mathrm{Ne}_0+{}^{1}_{0}n_1\rightarrow{}^{17}_{8}\mathrm{O}_1+{}^{4}_{2}\mathrm{He}_0$$
$${}^{19}_{9}\mathrm{F}_1+{}^{1}_{0}n_1\rightarrow{}^{16}_{7}\mathrm{N}_2+{}^{4}_{2}\mathrm{He}_0$$

in which nitrogen 16 is a new isotope of nitrogen. As in the earlier work on nitrogen, it is found that: (1) Neutrons effective in disintegration appear both to come directly from the source and to be scattered by nuclear impact prior to the disintegration. (2) Kinetic energy disappears in the process, or is (rarely) conserved. This kinetic energy decrement may be transformed into mass, if mass increases in the reaction, or into γ-rays; it may also excite the heavier product nucleus and later give rise to an artificial radioactivity. (3) The maximum, minimum and average kinetic energy for the neutrons which in our experiments have been found to disintegrate fluorine, neon and nitrogen are listed below in the table.

Carbon. Mass values obtained in positive ray work give 6.9 m.e.v. as the mass increase in the reaction:

$${}^{12}_{6}\mathrm{C}_0+{}^{1}_{0}n_1\rightarrow{}^{9}_{4}\mathrm{Be}_1+{}^{4}_{2}\mathrm{He}_0.$$

If the mass values are extremely accurate only neutrons with kinetic energy greater than about 6.9 m.e.v. can therefore disintegrate carbon. Of 6 disintegrations found among 6400 pairs of photographs with ethylene, only 1 involves a neutron which approximates this energy. The other disintegrations may be those of oxygen or nitrogen from the water vapor and trace of air in the chamber. Carbon has therefore not yet been disintegrated with certainty by neutrons. It is of interest that about 20 percent of the neutrons found in this work have extremely high velocities, so that their kinetic energy is from 13.6 to 15.1 m.e.v., and that the energy transformed into γ-rays rises as high as 10 m.e.v.

	No. of disintegrations	Kinetic Energy in m.e.v. Min.	Ave.	Max.
Nitrogen	(28)	1.9	5.4	16.1
Fluorine	(13)	1.9	6.7	13.2
Neon	(11)	3.1	10.6	15.1

I. Introduction

The first paper[1] of this series on the disintegration of light atoms by neutrons presented values related to the mechanics of the disintegration of twenty-eight nitrogen nuclei. Further experiments have since been carried out with deuterium, carbon, fluorine and neon. This paper gives the quantitative relations found for the disintegration of 11 neon and 13 fluorine nuclei.[2]

[1] Harkins, Gans and Newson, Phys. Rev. **44**, 529 (1933).

[2] For preliminary reports, see Harkins, Gans and Newson, Phys. Rev. **44**, 236, 945 (1933).